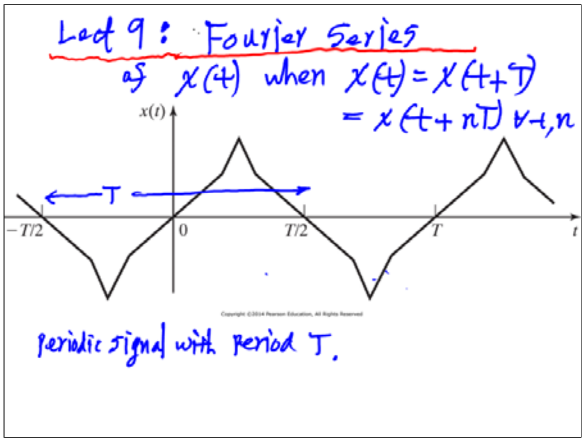
Lect 9: Fourier Series
of x(t) when x(t) = x(t+T)
= x(t+nT) ∀t,n
x(t)
T
−T/2
0
T/2
T
t
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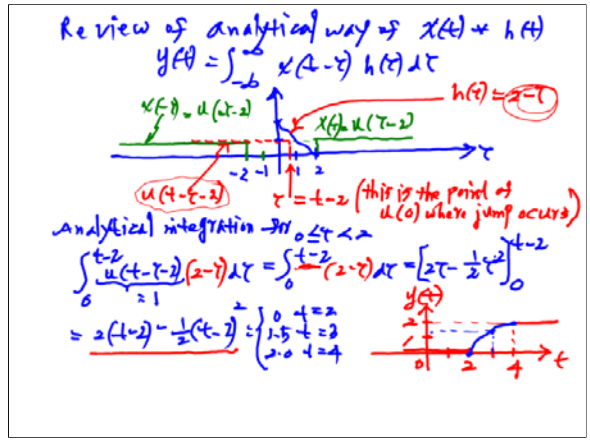
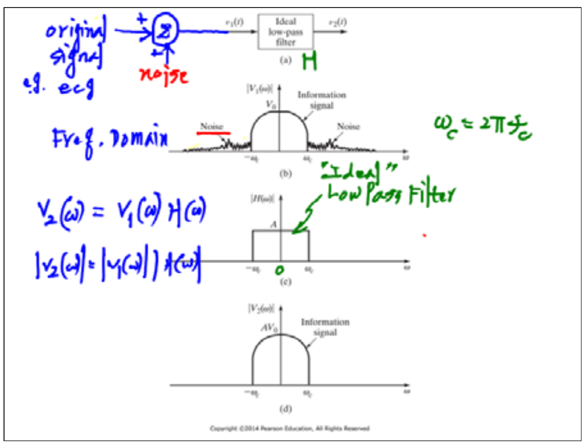
periodic signal with period T.


Review of analytical way of x(t) * h(t)
y(t) = ∫_{−∞}^{∞} x(t−τ) h(τ) dτ
h(t) = 2−τ
x(t) = u(t−2)
x(t) u(τ−2)
−2 −1 1 2
u(t−τ−2)
τ = t−2 (this is the point of u(0) where jump occurs)
Analytical integration for 0 ≤ τ < 2
∫_0^{t−2} u(t−τ−2)(2−τ) dτ = ∫_0^{t−2} (2−τ) dτ = [2τ − ½τ²]_0^{t−2}
= 1
= 2(t−2) − ½(t−2)² = 0 t=2; 1.5 t=3; 2.0 t=4
y(t)
2
1
0 2 4 t


original signal
e.g. ecg
noise
v1(t)
Ideal low-pass filter
v2(t)
(a)
H
|V1(ω)|
V0
Information signal
Noise
Noise
Freq. Domain
ωc = 2πfc
(b)
"Ideal" Low Pass Filter
V2(ω) = V1(ω) H(ω)
|V2(ω)| = |V1(ω)| H(ω)
|H(ω)|
A
0
(c)
|V2(ω)|
AV0
Information signal
(d)
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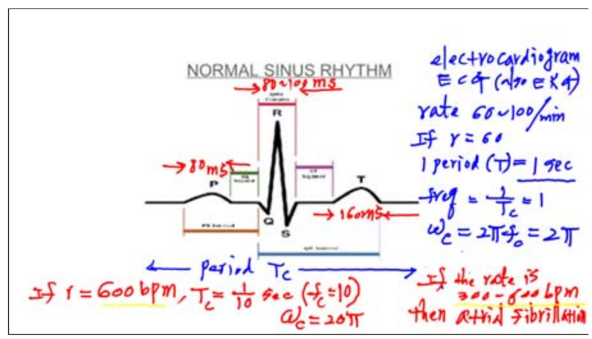
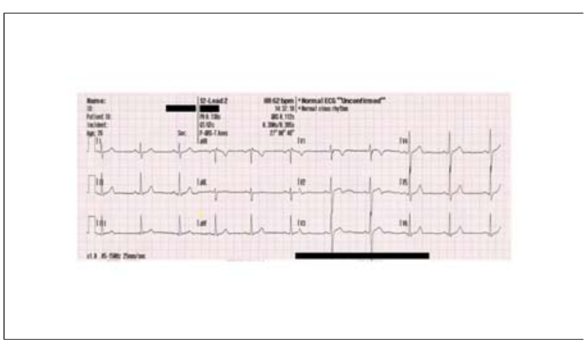
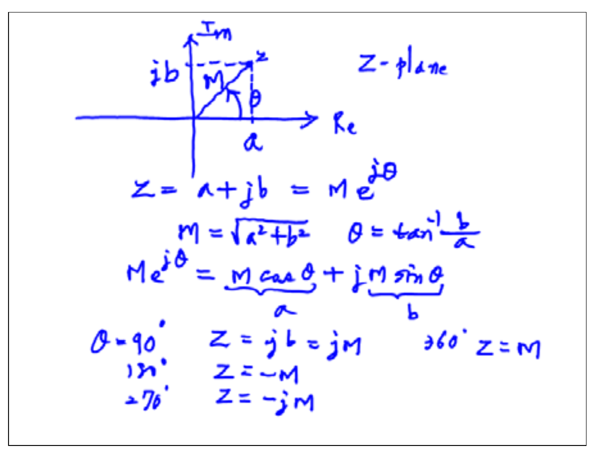

NORMAL SINUS RHYTHM
80~100 ms
R
P
T
Q
S
80ms
160ms
electrocardiogram
ECG (also EKG)
rate 60~100/min
If r = 60
1 period (T) = 1 sec
freq = 1/Tc = 1
ωc = 2πfc = 2π
period Tc
If r = 600 bpm, Tc = 1/10 sec (fc = 10)
ωc = 20π
If the rate is 300~600 bpm
then atrial fibrillation




Im
jb
z
M
θ
Re
a
Z-plane
Z = a + jb = M e^{jθ}
M = √(a² + b²)   θ = tan⁻¹ b/a
Me^{jθ} = M cos θ + j M sin θ
a
b
θ = 90°   Z = jb = jM
180°   Z = −M
270°   Z = −jM
360°   Z = M

$$e^{j\omega_0 t} \underset{\text{Euler's formula}}{=} \cos\omega_0 t + j\sin\omega_0 t \quad (1)$$

$$e^{-j\omega_0 t} = \cos(-\omega_0 t) + j\sin(-\omega_0)t$$
$$= \cos\omega_0 t - j\sin\omega_0 t \quad (2)$$

$(1)+(2) \rightarrow$
$$e^{j\omega_0 t} + e^{-j\omega_0 t} = 2\cos\omega_0 t \quad (3)$$
$$e^{j\omega_0 t} - e^{-j\omega_0 t} = 2j\sin\omega_0 t \quad (4)$$

from (3), (4)

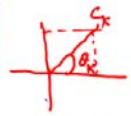
$$\boxed{\begin{aligned}\cos\omega_0 t &= \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)\\ \sin\omega_0 t &= \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)\end{aligned}}$$

$x(t)$ is periodic with period T_0

$$f_0 = \frac{1}{T_0} \quad \omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

Periodic signal $x(t)$ can be expressed by

$$\boxed{\underset{\text{real}}{x(t)} = \sum_{k=-\infty}^{\infty} C_k\, e^{jk\omega_0 t}}$$



C_K is a complex number

$$C_K = |C_K|\, e^{j\theta_K} = |C_K|(\cos\theta_K + j\sin\theta_K)$$
$$C_{-K} = |C_{-K}|\, e^{j\theta_{-K}}$$

$$C_K^* = |C_K|(\cos\theta_K - j\sin\theta_K)$$
$$= |C_K|\, e^{-j\theta_K}$$

$K=0$ $K=1$ $K=2$

$-k \;\cdots\; -4 \;\; -3 \;\; -2 \;\; -1 \;\; C_0 \;\; 1 \;\; 2 \;\; 3 \;\cdots\; k$

consider $k \neq 0$

$$C_{-K}\, e^{-jk\omega_0 t} + C_K\, e^{+jk\omega_0 t}$$
$$|C_{-K}|\, e^{j\theta_{-K}}\, e^{-jk\omega_0 t} + |C_K|\, e^{j\theta_K}\, e^{+jk\omega_0 t}$$

$|C_{-K}| = |C_K| \quad \theta_{-K} = -\theta_K$ for $C_{-K} = C_K^*$

$$\Rightarrow \quad |C_K|\Big(\underbrace{e^{-j(k\omega_0 t+\theta_K)} + e^{j(k\omega_0 t+\theta_K)}}_{2\cos(k\omega_0 t+\theta_K)}\Big)$$

$x(t) = |C_1|\left(e^{-j\omega_0 t} + e^{j\omega_0 t}\right)$

$$= 2|C_K|\cos(k\omega_0 t + \theta_K)$$

thus $$x(t) = C_0 + \sum_{k=1}^{\infty} 2|C_K|\cos(k\omega_0 t + \theta_K)$$

If $x(t) = 1\cos\omega_0 t$, then

$C_0 = 0$

$C_1 = \frac{1}{2} \quad \theta_1 = 0$

$C_2 = 0 \quad \theta_2 = 0$

$\vdots \quad 0 \quad \vdots \quad 0$

$x(t) = \cos\omega_0 t$

$= 0 + 2|C_1|\cos\omega_0 t + 0$

Calculation of C_K for given $x(t)$ periodic T_0

consider $$\int_0^{T_0 \text{ (period)}} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \int_0^{T_0}\left(\sum_{k=-\infty}^{\infty} C_K\, e^{jk\omega_0 t}\right) e^{-jn\omega_0 t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} C_K \int_0^{T_0} e^{j(k-n)\omega_0 t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} C_K \int_0^{T_0}\Big(\underset{1}{\cos(k-n)\omega_0 t} + j\,\underset{0}{\sin(k-n)\omega_0 t}\Big)\, dt$$

when $K=n$ Integration $\rightarrow C_K T_0$

Thus

$$\int x(t)\, e^{-jn\omega_0 t}\, dt \underset{k=n}{=} C_K T_0$$

$$\boxed{C_K \underset{n=K}{=} \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jk\omega_0 t}\, dt}$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_K\, e^{jk\omega_0 t}$$

⊠

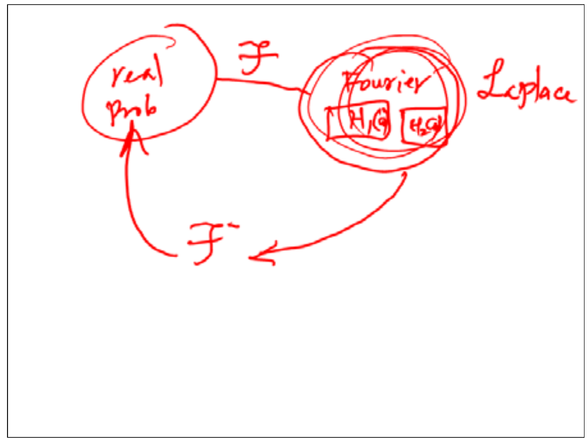
$$100 \times 10^{239} = 10^{2} \times 10^{239} = 10^{241}$$

$$\log_{10}\left(100 \times 10^{239}\right)$$

$$= \log_{10} 100 + \log_{10} 10^{239}$$

$$= 2 + 239 = 241$$

$$\boxed{10^{241}}$$



$$\boxed{\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = 10u(t)}$$

$$(s^2 + \cancel{6}s + 5) = 0 \qquad (s+5)(s+1) = 0$$

$$s = -5 \qquad s = -1$$

$$y_c = A e^{-5t} + B e^{-t}$$

$$y_p = 2 \qquad y(t) = 2 + A e^{-5t} + B e^{-t}$$

when R is absent (open)

$$s^2 + 5 = 0 \qquad (s + j\sqrt{5})(s - j\sqrt{5})$$

LC Tank for oscillation!

$$y(t) = A e^{-j\sqrt{5}t} + B e^{+j\sqrt{5}t}$$