EE103 Lecture 6

continuation of convolution

$\delta(t) \rightarrow \boxed{H} \rightarrow h(t)$

$x(t) \rightarrow \boxed{H} \rightarrow x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{\infty}^{-\infty} x(t-\tau')\, h(\tau')\,(-d\tau') = \int_{-\infty}^{\infty} x(t-\tau')\, h(\tau')\, d\tau'$$

$\tau' = t - \tau$

$d\tau' = -d\tau$

$\tau = t - \tau'$

$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = h(t) * x(t)$$

$$\boxed{x(t) * h(t) = h(t) * x(t)}$$

$x(t) \rightarrow \boxed{H_1}, \boxed{H_2} \rightarrow \Sigma \rightarrow y(t)$

$$y(t) = H_1(x(t)) + H_2(x(t)) = [H_1 + H_2](x(t))$$

$$= x(t) * h_1(t) + x(t) * h_2(t)$$

(distributive)

$$= x(t) * \underbrace{[h_1(t) + h_2(t)]}_{= h(t)}$$

$x(t) \rightarrow \boxed{H_1} \xrightarrow{H_1(x)} \boxed{H_2} \rightarrow y(t)$

$$y(t) = H_2 H_1(x(t)) = (x(t) * h_1(t)) * h_2(t)$$

$$= x(t) * \underbrace{[h_1(t) * h_2(t)]}_{= h(t) = h_2(t) * h_1(t)} = x(t) * h(t)$$

Associative

$y(t) = ?$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

Delay system

$\delta(t) \rightarrow \boxed{H} \rightarrow \delta(t - t_0)$

$h(t) = \delta(t - t_0)$

$\delta(t) \rightarrow \boxed{h(t)}_{t_0} \rightarrow \boxed{h(t)}_{t_0} \rightarrow \delta(t - 2t_0)$

$\delta(t) \rightarrow \boxed{h} \rightarrow \delta(t - t_0)$

$\quad\;\, \rightarrow \boxed{h}_{t_0} \rightarrow \delta(t - t_0)$

$x(t) \rightarrow \triangleright_{t_0} \rightarrow \triangleright_{t_0} \rightarrow x(t - 2t_0)$

$x(t) \rightarrow \boxed{H} \rightarrow y(t) = ? = \boxed{x(t - t_0)}$

$h(t) = \delta(t - t_0)$

check $\quad y(t) = x(t) * h(t)$

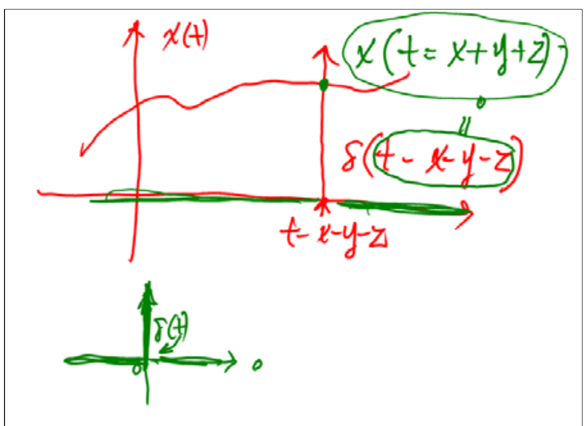
$$= \int_{-\infty}^{\infty} x(\tau)\, \delta(t - \tau - t_0)\, d\tau$$

$$x(t) * h(t) = \int x(\tau)\, h(t - \tau)\, d\tau$$

$$\delta[-(\tau - t + t_0)] = \delta(\tau - t + t_0)$$

$$= \int_{-\infty}^{\infty} x(t - t_0)\, \delta(t - \tau - t_0)\, d\tau$$

$$= x(t - t_0) \cdot 1 = \boxed{x(t - t_0)}$$



$x(t) \longrightarrow \boxed{\text{delay by 2}} \longrightarrow y(t)$

$x(t) = 4e^{-t}u(t)$

$y(t) = 4e^{-(t-2)}u(t-2)$

Mathematically

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} 4e^{-\tau}u(\tau)\,\delta(t-\tau-2)\,d\tau$$

$h(t) = \delta(t-2)$

$\neq 0$ when $\tau = t-2$

$$= 4e^{-(t-2)}u(t-2)\underbrace{\int_{-\infty}^{\infty}\delta(t-\tau-2)\,d\tau}_{=1}$$

$$= 4e^{-(t-2)}u(t-2)$$

$\delta(t) = \delta(-t)$ even

$\delta(t) = \text{Lim}\ \epsilon \to 0$

$\delta'(t) = -\delta'(-t)$ odd

$\delta''(t) = \delta''(-t)$ even

$\delta'''(t) = -\delta'''(-t)$ odd

$\vdots$

$$\boxed{\delta^{(k)}(t-t_0-\tau) = (-1)^k\,\delta^{(k)}(\tau-(t-t_0))}$$

For $y(t) = x(t) * \delta^{(k)}(t-t_0)$

$$= \int_{-\infty}^{\infty} x(\tau)\,\delta^{(k)}(t-\tau-t_0)\,d\tau$$

$$= (-1)^k \int_{-\infty}^{\infty} x(\tau)\,\delta^{(k)}(\tau-(t-t_0))\,d\tau$$

Kth deriv Property

$\delta^{(k)}(t-t_0-\tau) = (-1)^k\,\delta^{(k)}(\tau-(t-t_0))$

$$= (-1)^k(-1)^k x^{(k)}(t-t_0) = x^{(k)}(t-t_0)$$

$\delta(t) \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow \delta'(t)$

$x(t) \longrightarrow \boxed{\frac{d^k}{dt^k}} \longrightarrow x^{(k)}(t) = y(t)$

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system representation example

For $t \geq 0$

$$E\,u(t) = Ri + \frac{1}{C}\int i(\tau)\,d\tau + L\frac{di}{dt}$$

Taking $\frac{d}{dt}$ on both sides

$$E\,\delta(t) = R\frac{di}{dt} + \frac{1}{C}i + L\frac{d^2 i}{dt^2}$$