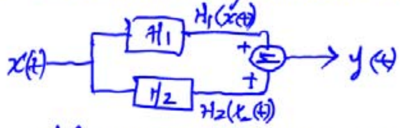
EE 103 Lecture 5

Tutor is Kevin Johnson xjohnson7@ucsc.edu

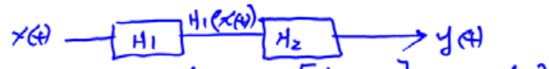
T & Th 12 ~ 1 pm

Sign up at http://eop.sa.ucsc.edu/OTSS/

Interconnected systems & convolution



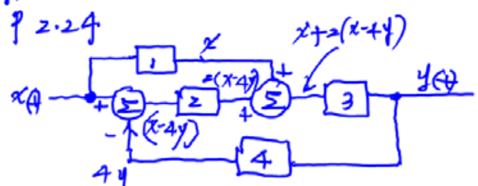
$y(t) = H_1(x(t)) + H_2(x(t)) = [H_1 + H_2](x(t))$



$y(t) = H_2[H_1(x(t))] = H_2H_1(x(t))$

e.g.

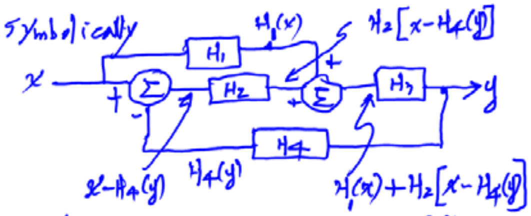
P 2.24



$y = 3(x + 2(x - 4y)) = 3x + 6x - 24y$

$y(1+24) = 9x$ $\quad \boxed{y = \frac{9}{25}x}$

Symbolically



$y = H_3[H_1(x) + H_2[x - H_4(y)]]$

$= H_3H_1(x) + H_3H_2(x) - H_3H_2H_4(y)$

$(1 + H_3H_2H_4)\,y = [H_3H_1 + H_3H_2](x)$

$y(t) = (1 + H_3H_2H_4)^{-1}[H_3H_1 + H_3H_2](x(t))$

$H_1 = 1,\ H_2 = 2,\ H_3 = 3,\ H_4 = 4$ $\quad y(t) = 25^{-1}(9)\,x(t) = \frac{9}{25}x(t)$

Convolution

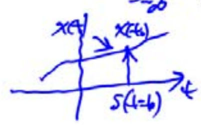
$\delta(t) \xrightarrow{H} h(t)$ impulse response

Recall $\int_{-\infty}^{\infty} \delta(t - t_0)\,dt = 1$

$\int_{-\infty}^{\infty} x(t)\,\delta(t - t_0)\,dt = \int_{-\infty}^{\infty} x(t_0)\,\delta(t - t_0)\,dt$

$= x(t_0)\int_{-\infty}^{\infty} \delta(t - t_0)\,dt$

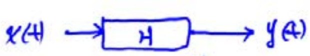
$= x(t_0)$



$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(\tau - t)\,d\tau$

$= \int_{-\infty}^{\infty} x(\tau)\,\delta(t - \tau)\,d\tau$

$x(t)$ is expressed as a function of $\delta(t)$!



$x(t) \xrightarrow{H} y(t)$

Assume H is LTI.

Impulse response $\delta(t) \xrightarrow{H} h(t)$

$\delta(t - t_0) \xrightarrow{H} h(t - t_0)$

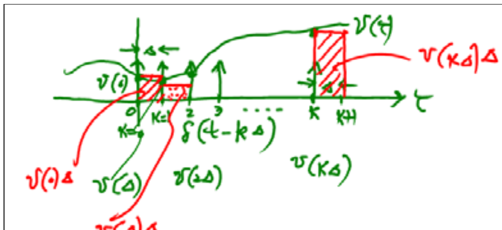
If $x(t) = \delta(t - k\Delta)\cdot\Delta \longrightarrow y(t)$

$= h(t - k\Delta)\Delta$

Suppose $\hat{x}_\Delta(t) = \sum_{k=-\infty}^{\infty} v(k\Delta)\,\delta(t - k\Delta)\Delta$

$\lim_{\Delta\to 0} \hat{x}_\Delta(t) = \lim_{\Delta\to 0} \sum_{k=-\infty}^{\infty} v(k\Delta)\,\delta(t - k\Delta)\Delta = x(t)$

$= \int_{-\infty}^{\infty} v(\tau)\,\delta(t - \tau)\,d\tau$



In the limit ($\Delta \to 0$) the area underneath

$v(t) = \lim_{\Delta\to 0} \sum_{k=-\infty}^{\infty} v(k\Delta)\,\delta(t - k\Delta)\Delta$

$= \boxed{\int_{-\infty}^{\infty} v(\tau)\,\delta(t - \tau)\,d\tau = v(t)}$

$$y_\Delta(t) = \sum_{k=-\infty}^{\infty} v(k\Delta)\, h(t-k\Delta)\, \Delta$$

$$y(t) = \lim_{\Delta \to 0} y_\Delta(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} v(k\Delta)\, h(t-k\Delta)\, \Delta$$

$$= \boxed{\int_{-\infty}^{\infty} v(\tau)\, h(t-\tau)\, d\tau}$$

$\delta(t) \longrightarrow \boxed{H} \longrightarrow h(t)$

$v(t) \longrightarrow \boxed{H} \longrightarrow \boxed{y(t) = \int_{-\infty}^{\infty} v(\tau)\, h(t-\tau)\, d\tau}$

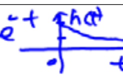
convolution

$$y(t) = v(t) * h(t)$$
$$= h(t) * v(t)$$

commutative property

example

$\delta(t) \longrightarrow h(t) = 2e^{-t}$

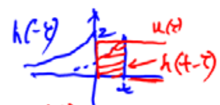


$v(t) = u(t)$

[Q] what is $y(t)$ for $\underset{=x(t)}{v(t)} = u(t)$

(Ans) $y(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$

$$= \int_{0}^{\infty} 1 \quad 2e^{-(t-\tau)}\, d\tau$$



$$= 2e^{-t} \int_0^t e^{\tau}\, d\tau$$

$$= 2e^{-t}\, e^{\tau}\Big|_0^t$$
$$= 2e^{-t}\left[e^{t}-1\right] = 2\left[1-e^{-t}\right]$$