Lecture #4
April 10, 2017

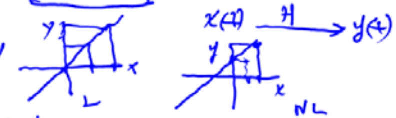
Read Sect 2.7, Sects 3.1 + 3.2

HW of Week 2

(1) Prob 2.15(a)
(2) prob 2-17
(3) prob 2-18(a)
(4) prob 2-29
(5) prob 3.3(a)
(6) prob 3.7
(7) Prob 3.9
(8) Prob. 3.24
(9) Prob 3.25(iv)

Continuous-time Linear Time-invariant systems

(i) Linearity



$x_1 \rightarrow y_1$ $\quad x_2 \rightarrow y_2$

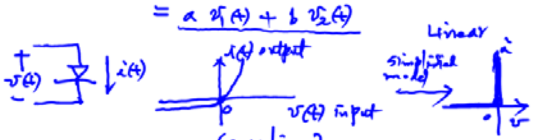
$$x = a x_1 + b x_2 \rightarrow y = a y_1 + b y_2$$
$$= H[a x_1 + b x_2] = H[a x_1] + H[b x_2] = a H(x_1) + b H(x_2)$$
$$= a y_1 + b y_2$$

e.g.)

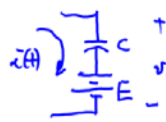
$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

$$y(t) = \frac{1}{C}\int_{-\infty}^{t}\left[a\, i_1(\tau) + b\, i_2(\tau)\right] d\tau$$
$$= \frac{1}{C}\left[a\int_{-\infty}^{t} i_1(\tau)\,d\tau + b\int_{-\infty}^{t} i_2(\tau)\,d\tau\right]$$
$$= a\, v_1(t) + b\, v_2(t)$$

Linear



(nonlinear)



$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau + E$$

(nonlinear)

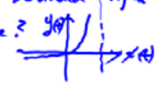
ii) Causality

output $y(t)$ at $t = t_0$ depends only on $x(t)$ $t \le t_0$ (not $t > t_0$)

$$y(t_0) = \frac{1}{C}\int_{-\infty}^{t_0} i(t)\,d\tau + E \quad (\text{not } t_0 + \text{any})$$

iii) BIBO stability (Bounded Input Bounded output)

Is Diode BIBO stable?



not BIBO stable.

iv) Time Invariance

A time shift in $x(t)$, say $x(t-t_0)$, results in the same time shift in $y(t)$, i.e. $y(t-t_0)$ for all t_0 for all t.

v) Memory

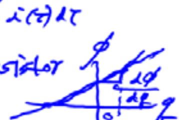
At $t = t_0$, $y(t_0)$ depends on $x(t)$ $t \le t_0$.

e.g.) capacitor $v(t_0) = \frac{1}{C}\int_{-\infty}^{t_0} i(\tau)\,d\tau$

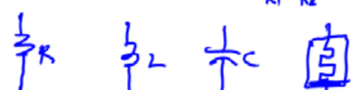
e.g.) Memristor = Memory Resistor

$\phi(t) = f(q(t))$

$$\frac{d}{dt}\phi(t) = v(t) = \frac{df(q)}{dq}\frac{dq}{dt} = i(t) \quad \left(v = \left[\frac{df(q)}{dq}\right] i\right)$$



$$y(t) = v(t)\frac{R_2}{R_1 + R_2} \underset{R_1 = R_2}{=} \frac{1}{2} v(t)$$

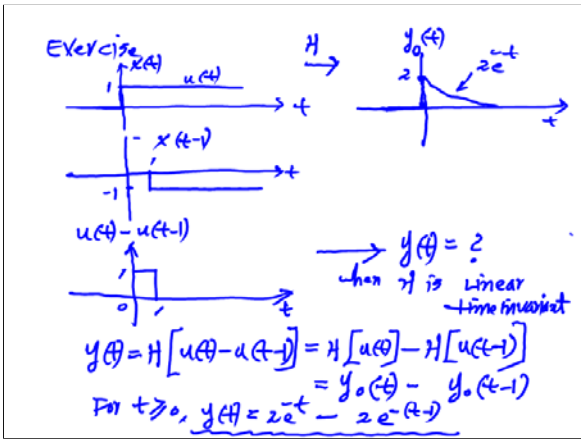


e.g.) Is $y(t) = x(t) + x(t-3)$ does it have a memory?

Ans. YES

$y(t_0)$, $t_0 = 5$: $y(5) = x(5) + x(5-3) = x(5) + x(2)$

e.g.) $y(t) = (t-3)\,x(t)$ Memoryless


Exercise
x(t)
u(t)
1
t
H
$y_0(t)$
2
$2e^{-t}$
t
x(t-1)
1
t
-1
u(t) - u(t-1)
1
0
1
t
$y(t) = ?$
then H is Linear
time invariant
$y(t) = H[u(t) - u(t-1)] = H[u(t)] - H[u(t-1)]$
$= y_0(t) - y_0(t-1)$
For $t \geq 0$, $y(t) = 2e^{-t} - 2e^{-(t-1)}$