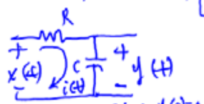
EE 103 Lect #1

$x \longrightarrow \boxed{H} \longrightarrow y \qquad y = H(x)$



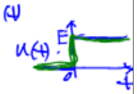
assume $y(0)=0$

$i(t) = C\frac{dy(t)}{dt}$

$x(t) = R\,i(t) + y(t)$

$= RC\frac{dy(t)}{dt} + y(t)$

Let's find $y(t)$ for $x(t) = E\,u(t)$



$E = RC\frac{dy(t)}{dt} + y(t)$

Taking Laplace Transform on both sides

$$\frac{E}{s} = RC\left[sY(s) + y(0)\right] + Y(s) = (RCs+1)Y(s)$$

$y(0) = 0$

$$Y(s) = \frac{E}{s(RCs+1)} = \frac{E}{RC}\frac{1}{s\left(s+\frac{1}{RC}\right)}$$

$$= \frac{E}{RC}\underbrace{\left[\frac{A}{s} + \frac{B}{s+\frac{1}{RC}}\right]}$$

$$\frac{As + \frac{A}{RC} + Bs}{s\left(s+\frac{1}{RC}\right)} = \frac{1}{s\left(s+\frac{1}{RC}\right)}$$

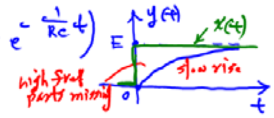
thus $A + B = 0 \quad A = -B$ and $\frac{A}{RC} = 1$

$A = RC, \ B = -RC$

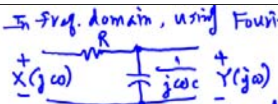
thus $Y(s) = \frac{E}{RC}\left[\frac{RC}{s} - \frac{RC}{s+\frac{1}{RC}}\right] = E\left[\frac{1}{s} - \frac{1}{s+\frac{1}{RC}}\right]$

$\downarrow \mathcal{L}^{-1}$

$$y(t) = E\left(1 - e^{-\frac{1}{RC}t}\right)$$

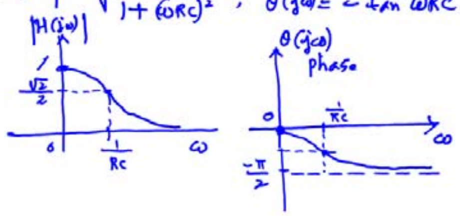


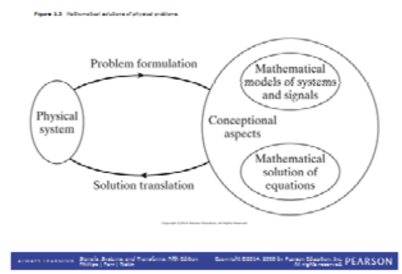
In freq. domain, using Fourier Transform



$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}, \quad \theta(j\omega) = -\tan^{-1}\omega RC$$





A quick trip to the methods of Laplace & Fourier

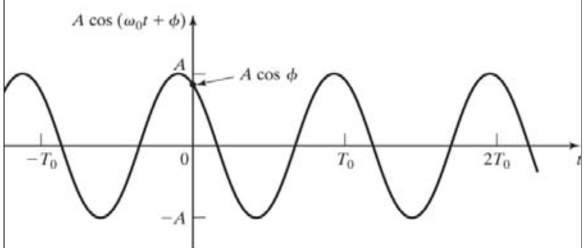
Details of these methods will be covered later on.

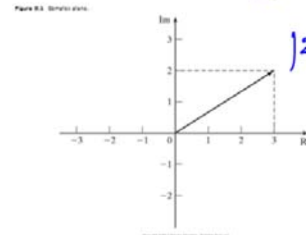
HW is assigned for each week

this week's HW assignment:

HW is not collected/corrected.
Each student is responsible to solve them correctly (using answers to be provided)

Following Monday, we will have a 15 min. Quiz
– one problem chosen from the assigned HW probs.
(parameters may change, but not problems)

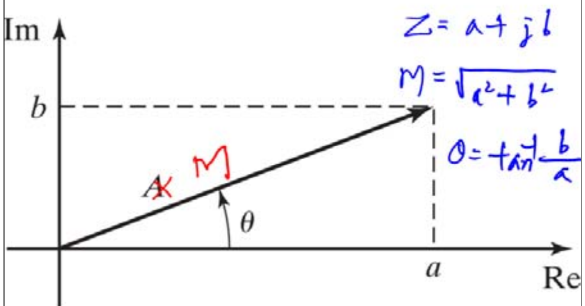




$Z = 3 + j2$

$|Z| = M = \sqrt{3^2 + 2^2} = \sqrt{13}$

$\phi = \tan^{-1}\frac{2}{3}$



$Z = a + jb$

$M = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}\frac{b}{a}$

$$Z = (a+jb)(c+jd) = M_1 e^{j\theta_1} \cdot M_2 e^{j\theta_2} = M_1 M_2 e^{j(\theta_1+\theta_2)}$$

Euler's formula

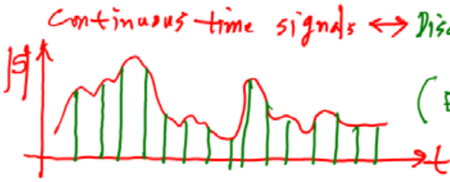
$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$
$$e^{j\theta} + e^{-j\theta} = 2\cos\theta \Rightarrow \boxed{\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)}$$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$(a+jb)^c = \left(M e^{j\theta}\right)^c = M^c e^{j\theta c} = M^c(\cos\theta c + j\sin\theta c)$$

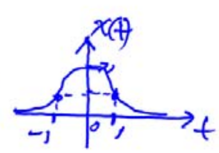
Continuous time signals $\leftrightarrow$ Discrete time signals

(EE 153)

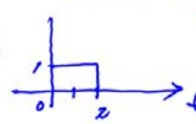


Signal synthesis

(e.g.) $x(t) = \frac{2}{1+t^2}$



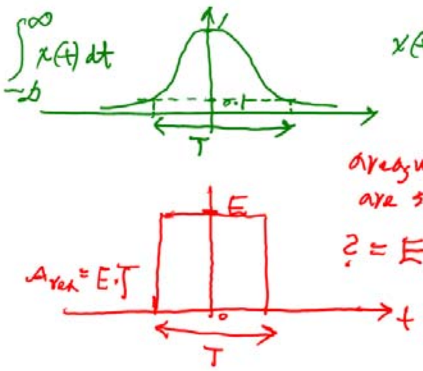
$x_2(t)$



$$x_2(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \le t < 2 \\ 0 & t \ge 2 \end{cases}$$

$\int_{-\infty}^{\infty} x(t)\,dt$

$x(t) = \frac{1}{1+t^2}$



area underneath are same

$? = E$

$$\int_{-\infty}^{\infty} \frac{1}{1+t^2}\,dt = ET$$

$$E = \frac{1}{T}\underbrace{\int_{-\infty}^{\infty} \frac{1}{1+t^2}\,dt}_{\tan^{-1} t\,\big|_{-\infty}^{\infty}}$$

$$= \frac{1}{T}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \frac{\pi}{T}$$

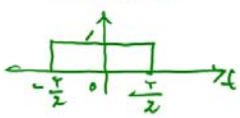
$x(t)$

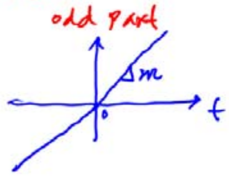
$x_e(t) = ?$

even part

$x_o(t) = ?$

odd part





For a given signal $x(t)$

$x_e(t) \triangleq \frac{1}{2}[x(t) + x(-t)]$

$x_o(t) \triangleq \frac{1}{2}[x(t) - x(-t)]$

Properties

$x_e(t) = x_e(-t)$

$x_o(t) = -x_o(-t)$