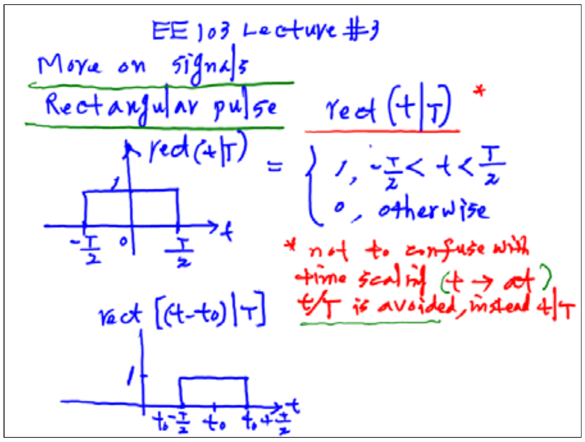
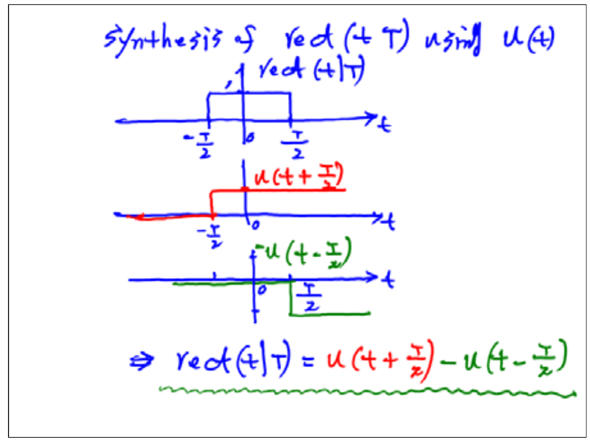
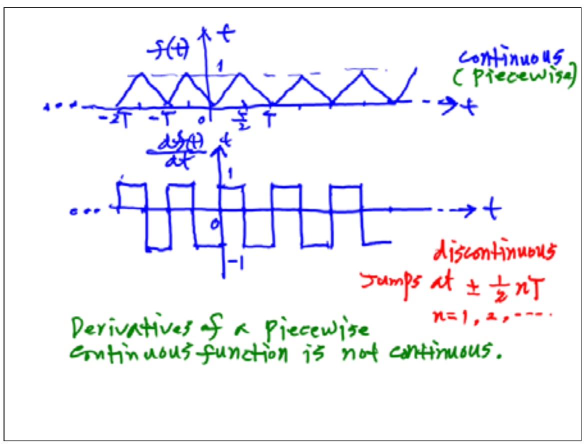
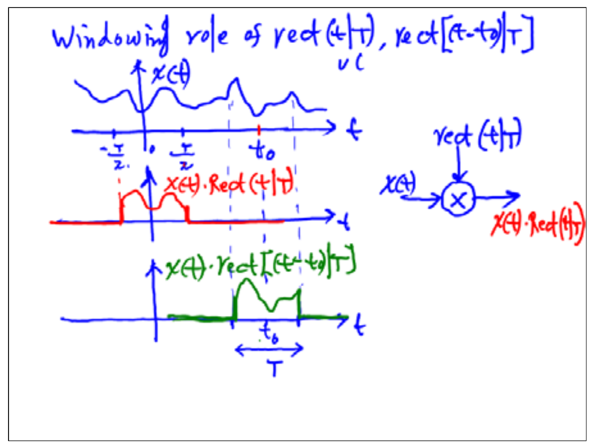
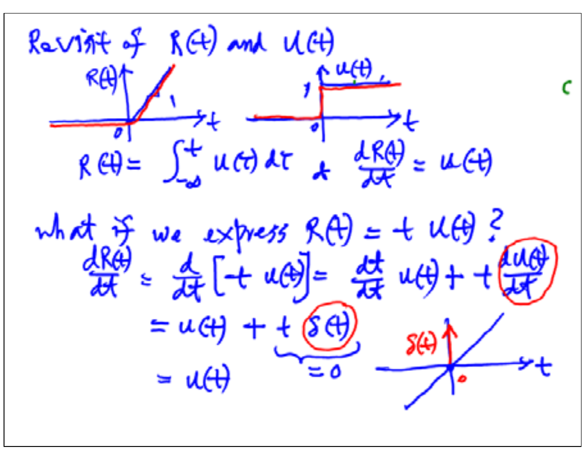
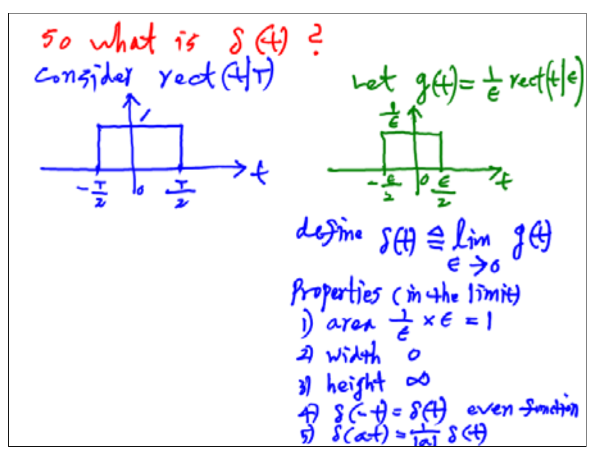

EE 103 Lecture #3
More on signals
Rectangular pulse
rect (t|T) *
rect (t|T) = { 1, -T/2 < t < T/2 ; 0, otherwise
* not to confuse with time scaling (t → at)
t/T is avoided, instead t|T
rect [(t-t0)|T]
t0-T/2 t0 t0+T/2


Synthesis of rect (t T) using u(t)
rect (t|T)
u(t + T/2)
u(t - T/2)
⇒ rect(t|T) = u(t + T/2) - u(t - T/2)


f(t)
continuous (piecewise)
df(t)/dt
discontinuous
Jumps at ± 1/2 nT
n=1, 2, ---
Derivatives of a piecewise continuous function is not continuous.


Windowing role of rect(t|T), rect[(t-t0)|T]
x(t)
x(t)·Rect(t|T)
rect(t|T)
x(t)·Rect(t|T)
x(t)·rect[(t-t0)|T]
t0
T


Revisit of R(t) and u(t)
R(t) = ∫_{-∞}^{t} u(τ) dτ & dR(t)/dt = u(t)
what if we express R(t) = t u(t) ?
dR(t)/dt = d/dt [t u(t)] = dt/dt u(t) + t du(t)/dt
= u(t) + t δ(t)
= u(t)
= 0
δ(t)


So what is δ(t) ?
Consider rect(t|T)
Let g(t) = 1/ε rect(t|ε)
define δ(t) ≜ lim_{ε→0} g(t)
Properties (in the limit)
1) area 1/ε × ε = 1
2) width 0
3) height ∞
4) δ(-t) = δ(t) even function
5) δ(at) = 1/|a| δ(t)

$\delta(t) = 0 \quad$ for $t \neq 0$

$\delta(t-t_0) = 0 \quad$ for $t \neq t_0$

$\int_{-\infty}^{\infty} \delta(t-t_0)\,dt = 1 \quad$ for $-\infty < t_0 < \infty$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = \int_{-\infty}^{\infty} f(t_0)\,\delta(t-t_0)\,dt = f(t_0)\int_{-\infty}^{\infty}\delta(t-t_0)\,dt = \underline{f(t_0)}$$

$$e^{-3t}\,\delta(t-1) = e^{-3}\,\delta(t-1)$$

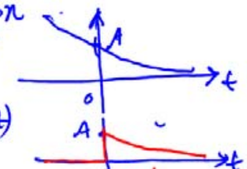
$$e^{\sin \pi t}\,\delta(t-5) = e^{\sin 5\pi}\,\delta(t-5) = \delta(t-5)$$

$$\int_{-3}^{2} f(t)\left[2+\delta(t+1) - 3\delta(t-2) + 10\,\delta(t-3)\right]dt = 2\int_{-3}^{2} f(t)\,dt + f(-1) - 3f(2) + \boxed{0}$$

Exponential Function

$x(t) = A\,e^{-\alpha t}$

$x(t) = A\,e^{-\alpha t}\,u(t)$



what about for $t = -\infty$? $\quad e^{-\alpha t} = \infty$!

Ill defined !

$x(t) = \text{Sinc}(t) = \dfrac{\sin t}{t}$

what is $\text{Sinc}(0)$?

For $t = 0$
$$\frac{\left.\dfrac{d(\sin t)}{dt}\right|_{t=0}}{\dfrac{dt}{dt} = 1} = \cos t\Big|_{t=0} = 1$$

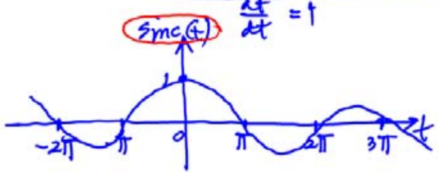


TABLE 2.3 Properties of the Unit Impulse Function

1.	$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)\,dt = f(t_0),\ f(t)$ continuous at $t = t_0$
2.	$\int_{-\infty}^{\infty} f(t-t_0)\delta(t)\,dt = f(-t_0),\ f(t)$ continuous at $t = -t_0$
3.	$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0),\ f(t)$ continuous at $t = t_0$
4.	$\delta(t-t_0) = \dfrac{d}{dt}u(t-t_0)$
5.	$u(t-t_0) = \int_{-\infty}^{t}\delta(\tau-t_0)\,d\tau = \begin{cases}1, & t > t_0\\ 0, & t < t_0\end{cases}$
6.	$\int_{-\infty}^{\infty}\delta(at-t_0)\,dt = \dfrac{1}{\lvert a\rvert}\int_{-\infty}^{\infty}\delta\left(t-\dfrac{t_0}{a}\right)dt$
7.	$\delta(-t) = \delta(t)$

$\sin \omega t \leftrightarrow \delta(t)$

$\sin^2 \omega t \leftrightarrow [\delta(t)]^2 \quad \times$ no meaning

$\dfrac{d}{dt}(\sin \omega t) \leftrightarrow \dfrac{d}{dt}\,\delta(t)$?

$= \omega \cos \omega t$

$$\lim_{\epsilon\to 0}\frac{1}{\epsilon}\left[\delta\left(t+\frac{\epsilon}{2}\right) - \delta\left(t-\frac{\epsilon}{2}\right)\right]$$

$$\int_{-\infty}^{\infty} x(t)\,\delta'(t)\,dt = \lim_{\epsilon\to 0}\frac{1}{\epsilon}\int_{-\infty}^{\infty} x(t)\left[\delta\left(t+\frac{\epsilon}{2}\right) - \delta\left(t-\frac{\epsilon}{2}\right)\right]dt$$
$$= \lim_{\epsilon\to 0}\frac{1}{\epsilon}\left[x\left(-\frac{\epsilon}{2}\right) - x\left(\frac{\epsilon}{2}\right)\right] = \boxed{-x'(0)}$$

In general

$$\boxed{\int_{t_1}^{t_2} x(t)\,\delta^{(k)}(t-t_0)\,dt = \begin{cases}(-1)^k\,x^{(k)}(t_0), & t_1 \le t_0 \le t_2\\ 0, & \text{otherwise}\end{cases}}$$

where $\delta^{(k)}(t) = \dfrac{d^k}{dt^k}\,\delta(t)$

$x^{(k)}(t) = \dfrac{d^k}{dt^k}\,x(t)$

(e.g.) $\int_{-3}^{5} t^5\,\delta^{(3)}(t-4)\,dt = (-1)^3\,60\,t^2\,(t=4)$

$= -960$

$[t^5(t) \to 5t^4 \to 20t^3 \to 60t^2]$