EE 103 Lecture 10

HW for wk 4

1. prob 4.23 (a)
2. prob 4.29
3. prob 5.1 (a) & (b)
4. prob 5.7 (b)
5. prob 5.8

Read chap 4 (rest) & chap 5 (5.1 & 5.2)

A signal $x(t)$ is periodic with period T if $x(t) = x(t+T)$ for all t

Then $x(t) = x(t+kT)$ for all integer k

$\omega_0 = 2\pi f_0 = 2\pi \frac{1}{T_0}$ $\quad \boxed{\omega_0 T_0 = 2\pi}$

T_0 = minimum value of $T > 0$ that satisfies $x(t) = x(t+T)$

$x(t) = A_1 \cos(k_1 \omega_0 t + \theta_1) + B_1 \sin(k_2 \omega_0 t + \theta_2)$ is periodic for all k_1, k_2 integers

counter example

$x(t) = \sin t + \cos \pi t$ (not periodic)

$1t : \pi t = 1 : \pi$ not integer ratio!

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \qquad \begin{matrix} 1 & k\omega_0 \\ \pi & l\omega_0 \end{matrix}$$

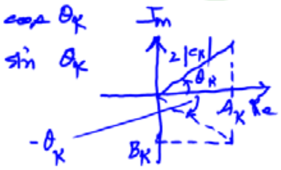
$z = |z| e^{j\theta}$

$$= c_0 + \sum_{k=1}^{\infty} 2|c_k| \cos(k\omega_0 t + \theta_k)$$

$$= c_0 + \sum_{k=1}^{\infty} (A_k \cos k\omega_0 t + B_k \sin k\omega_0 t)$$

where $A_k = 2|c_k| \cos \theta_k$

$B_k = -2|c_k| \sin \theta_k$



Calculation of c_k

$$c_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} \, dt \qquad e^0 = 1$$

$$c_0 = \frac{1}{T_0} \int_{T_0} x(t) \, dt$$

$\sin 1t$

$\omega_0 = 2\pi f_0 = 1$

$f_0 = \frac{1}{2\pi}$

$T_0 = 2\pi$

$\cos \pi t \quad \omega_0 = 2\pi f_0 = \pi$

$f_0 = \frac{1}{2}$

$T_0 = 2$

Euler

$e^{+jk\omega_0 t} = \cos k\omega_0 t + j \sin k\omega_0 t$

$e^{-jk\omega_0 t} = \cos k\omega_0 t - j \sin k\omega_0 t$

$\Sigma / 2 \quad = \cos k\omega_0 t = \frac{e^{+jk\omega_0 t} + e^{-jk\omega_0 t}}{2}$

$\frac{e^{+jk\omega_0 t} - e^{-jk\omega_0 t}}{2j} = \cancel{2j} \sin k\omega_0 t$

$$\cos(k\omega_0 t + \theta_k)$$
$$= \cos k\omega_0 t \cdot \cos\theta_k - \sin k\omega_0 t \, \sin\theta_k$$
$$(\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta)$$
$$2|c_k|\cos\theta_k = A_k$$
$$-2|c_k|\sin\theta_k = B_k$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{+jk\omega_0 t}$$
$$= \cdots + c_{-k} e^{-jk\omega_0 t} + \cdots + c_0 + \cdots + c_k e^{+jk\omega_0 t} \cdots \quad (k>0)$$

for k case $\quad c_{-k} e^{-jk\omega_0 t} + c_k e^{+jk\omega_0 t}$

$c_{-k} = c_k^*$ $\quad |c_{-k}| = |c_k|$ $\quad \angle c_{-k} = -\angle c_k$, $\theta_{-k} = -\theta_k$

$(c_{-k})^* = (c_k^*)^* = c_k$

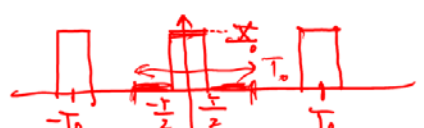
$$|c_k| e^{-j\theta_k} e^{-jk\omega_0 t} + |c_k| e^{+j\theta_k} e^{+jk\omega_0 t}$$
$$2|c_k| \, \frac{e^{-j(k\omega_0 t+\theta_k)} + e^{+j(k\omega_0 t+\theta_k)}}{2}$$

(example 1)

Find c_k

$$c_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$
$$= \frac{1}{T_0} \cdot 1 = \frac{1}{T_0} \quad \forall k$$
$$c_k = \frac{1}{T_0}\int_{mT_0}^{(m+1)T_0} x(t)\, e^{-jk\omega_0 t}\, dt =$$

$$e^{-j k_m (2\pi)}$$
$$\int_0 \delta(t)\, y(t)\, dt$$



$T < T_0$

$$c_k = \frac{1}{T_0}\int_{-\frac{T}{2}}^{\frac{T}{2}} X_0\, e^{-jk\omega_0 t}\, dt$$
$$= \frac{X_0}{T_0}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0} \Bigg|_{-\frac{T}{2}}^{\frac{T}{2}} \right]$$
$$= \frac{X_0}{T_0(-jk\omega_0)}\left[e^{-jk\omega_0 \frac{T}{2}} - e^{-jk\omega_0(-\frac{T}{2})} \right]$$